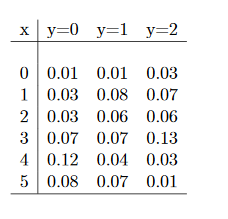
**STAT603 Homework 8**

**3** Consider a ferry that can carry both cars and campers across a waterway. Each trip costs the  
owner approximately $10. The fee for cars is $3 and the fee for campers is $9. Let X and Y  
be the number of cars and campers, respectively on a given trip. Suppose the probabilities  
for different values of X and Y are given by the following table.



(a) The revenue for the ferry is R = 3X + 9Y. Find the possible values of R and the associated  
probabilities.  
(b) Compute the expected revenue E(R) and the expected profit for the ferry owner. What is  
the standard deviation of the profit?Use Python to do the calculations using scipy

**4** A manufacturing company uses an acceptance plan on items from a production line before  
they are shipped. Boxes of 25 items are readied for shipment, and a sample of 3 items is  
tested for defectives. If any defectives are found, the entire box is sent for 100% screening. If  
no defectives are found, the box is shipped. Identify the parameters of the Hypergeometric  
distribution and calculate these probabilities.  
(a) What is the probability that a box, containing 4 defectives will be shipped?

**Solution:** Population Size(N) = 25

Population(K) = 4 for (a) and 1 for (b). Sample size(n) = 3. Number of defective items in sample k, we will find out. By Hypergeometric formula

P (X = k) = Kick\*(N-K) C(n-k) **/**NC, if the box is shipped, k =0

P (X = 0) = 4C0 \* (25 – 4) C (3-0)/25C3 = 1 \* 21C1/25C3 = 1330/2300 = **0.587.**  
(b) What is the probability that a box containing only 1 defective will be sent back for screening?

**Solution:** By Hypergeometric formula

P (X = k) = Kick\*(N-K) C(n-k) **/**NC 1C0\*(25-3) C (3-0)/25C3 = 1771/2300 = 0.777

For k>=1, P (At least 1 is defective) = 1 – 0.777= **0.23**

**4.2** Define a random variable X by the following procedure. Draw a card from a standard deck of playing cards. If the card is knave, queen, or king, then X =11. If the card is an ace, then X =1; otherwise, X is the number of the card (i.e., two through ten). Now define a second random variable Y by the following procedure. When you evaluate X, you look at the color of the card. If the card is red, then Y =X -1; otherwise, Y=X + 1.

(a) What is P(X<=2)?

**Solution:** P(X<=2) = P (Ace or 2) = P(Ace)+P (2) = 4/52 + 4/52 = 8/52 = **2/13.**   
(b) What is P(X>=10)?

**Solution:** P(X>=10) = P (X can be 10, 11, 12, or 13 (king, knave, or queen))

P (10, J, Q, K) = 3\*4/52 = 12/52 = **4/13**  
(c) What is P(X>=Y)?

**Solution:** P(X>=Y) = P (Y = X + 1) = 1/2 as both red and black cards are equally likely here.  
(d) What is the probability distribution of Y - X?

**Solution:** The PDF for X is first obtained here as:

P (X = 1) = P (X = 2) .... = P (X = 10) = 1/13 and P(X=11) = 3/13.

Now, for red cards, we have Y = X - 1 that is Y - X = -1. For non-red, we have: Y - X = 1Therefore, we have here:   
P (Y - X = -1) = 1/2 = 0.5   
P (Y - X = 1) = 1/2 = 0.5  
(e) What is P(Y>=12)?

**Solution**: P (Y >= 12) = P (Y = 12) as Y cannot be more than 12.

= P (X = 11) P (C () = (3/13) \*05= **3/26**

**4.3** Define a random variable by the following procedure. Flip a fair coin. If it comes up heads, the value is 1. If it comes up tails, roll a die: if the outcome is 2 or 3, the value of the random variable is 2. Otherwise, the value is 3.

(a) What is the probability distribution of this random variable?

**Solution:**  Let X be the random variable,

If head up then X =1, P(X) = ½

If tail up then roll a die => outcome is 2 or 3 =>X=2

P(X=2) = ½ \* 2/6 = 1/6

If outcome not 2 or 3 =>X =3

P(X=3) = ½ \* 4/6 = 1/3

So, probability distribution of X is represented by below table

|  |  |  |  |
| --- | --- | --- | --- |
| X | 1 | 2 | 3 |
| P(X) | ½ | 1/6 | 1/3 |

(b) What is the cumulative distribution of this random variable?

**Solution:**  For cumulative distribution Fix(X) = P(X<=x)

For x<1, Fix(X) = P(X<=x) = 0

For 1<=x<2, Fix(X) = P(X<=x) = P(X=1) = ½

For 2<=x<3, Fix(X) = P(X=1) +P(X=2) = 1/6+1/3 = 2/3

For 3>=x, Fix(X) = P(X<=x) = P(X=1) +P(X=2) +P(X=3) = ½+1/6+1/3 = 1

So, cumulative distribution of this random variable is resented by below table

|  |  |
| --- | --- |
| x<=1 | 0 |
| 1<=x<2 | 1/2 |
| 2<=x<3 | 2/3 |
| X>=3 | 1 |